

Risk, Ambiguity and Decision

Daniel Ellsberg

with an introduction by Isaac Levi

x-Liima

? 208n - Enigma - Garage E/T

certainty - Primer
82

VIENNA CONF. NOTES
MAY 2010

x "Paradox"

ME of more

(only 18th)

and not - (17th)

50-51

gain/loss: 0: fine
82

Enigma x1, 18 gains, 16

50 96, 14
39 16

See this Principle
42 (W3? W3 "s-t"?)

100 "ready acceptance"

arbitrary # number of
"paradoxes" 51 is note

def: 1, 11, 16, 183 196 (vector:
p.o.)

Chipm

14/43

201.

102
= 106000

(190) MY criterion

Restricted Bayes-Humming

(191/192)

(203)

190 - Humming/Bhavelle!

195

x Altais 255

not only, "Primer"

x (Definite judgments of 96, 134-35
relative "prob." are not possible - either to
state (intuitively) or infer from choices
(e.g. "="

"not >" and "not <" does not imply "="
no, neither $\leq$ nor $\geq$!

In 2 wms, $R_I \neq R_{II}$, nor is $R_I \neq R_{II}$

since "not R_I " is not $\leq$ not- R_{II}

x Kilmer, R and Y (or B)
138-39 R "looks" > Y, and $\bar{R}$ looks > $\bar{Y}$

K/T

compared with
264

(263) ERRORS (194?) + or X (?)?

litho. 255 allis date

my error
(86)

Error? hypo

OTES

X W. Dutch boat
caterpillar
70-71-72, 74
(79-79) 122

DUTCH BOAT

86 (87)

137 -100

2 ?

(81)

K/T (47)

209

71 picnic

84 Y

XHY

ALLAS-SW 260

4-COLOR URN

199-200-203

(263) ? 140?

183 set, latter

def.

Po - p. 64

Po 67 = 1 - qo

RAMSAY: (81)

1 - 261

Berpostli:

54-63

44-51

X Sample

wrong

Po, qo 2/7 "probs" 65
68

Po = 1 - qo

Po + qo = 1

Po + qo = 1

Po = Po

Caterpillar 75-76

44 77 (Ramsay) 20

79 - Sample

87-88 Sample

87 - Sample

97 Sample (Ramsay)

internal "figure of belief": 71-72, 85
96

two observations mixed - 69, 89

145 (86)

DE 3-color urn 43, 137

("HOPEFUL")

X Pao - ambiguity

78-79 (143) X

(6) 264

44

134-35 171

138 X (199) 197

217 200

wishful:

(2) seeing more

ambiguity, and

(6) by label;

or...

Burattini 1738 p. 45

Ramsay: degree of belief 81

X Requirement for odds

to be definite and constant

(Ramsay) 83

(84) (86) (140) X

#

"introduction of ambiguity"

145-46

"consistent" 87, 86, 98

20 176 (Good)

X 138, 134 (43)

"non-experiments"

(1953-62) Brannon-

44 - Brannon 1961

Winters
1960

STUDIE

edited by
ROBERT N
PELEGRINO
AT HARVARD

145 - ERROR

Supers

265 183
58 194
183 196 158
109? 206 177
71 208 18
84 243 19
97 248 22
(green) 252
2 263 an
31 157?

54
104 Beyond
166 R/T
(Error) A GARLAND
200-01 Content
(all

203
ERROR 256
204? subject
223 249

Savage uncorrelated

211

legibility of prob
265

2 observations/units
(at a min.) required
to establish "prob."

88.4 82 65-68

81

67

145

83

95

69

86

140-141

"Paradox" 188

51

217

48

181 - intuitive notion
an "almost essential"

in dealing with
behavior violating action

(O, with underlying
intensity?

(O) from
pain / loss)

S. L. P. / P2

defined

42

182 - verbal specification

208 9x1 against Greeley
(unknown 10-color unit)

216 my theory
76 184-25
190-

Blake / Lawrence
reliability / multiple
(51)

138-39 All 1K "Landscape" > 1, 2, 3

intolerance of ambiguity
1960

STUDIES IN PHILOSOPHY

favorable possibilities

199 ~~200~~ 78-79 171

217

(44)

203

206

134-35

(174)

258

138

137n

258n

264

(143)

edited by K/T: 157

ROBERT NOZICK

PELLEGRINO UNIVERSITY PROFESSOR
AT HARVARD UNIVERSITY

195 ERROR?

(Sofar)

Kerrigan K/T

265 183

205n

(58) 194 error

207

193 196 158

217

109? 206 172

(249) 251

71 208 180

258 allain

34 243 192

67

77 248 249

(47)

main 252

2 263 error

(137)

21 137?

135? PT

24

140

24

Beyond this book:

K/T (249) 251

145

A GARLAND SERIES

intol. of

200-01 Culture of America

ambig.

(allain) 247, 256

203

ERROR

256 "objection"

204?

objection ambig.

223

-249 253

Quote (Vinson)

x1 183-95

1 191

11 (194)

16 (SA) 198

29 (197)

242/B-4

199-200

N-COLOR

URA

38 201

141 202

(203)

(143)

206

97/165

208

(208)

173

212

188

(215)

(258)

216

(262)

249

258

258

Thyge

WRONG

wrong (97)

wrong

wrong

210-11

217

(206)

(262)

(6. allain)

58 Antich book

51n

12-13 & odd

3 & double

39

70

74-75-59

106

108

my resolution

244 (p. 37)

245

16

50-51

48

16

(145)

212

(us. Kaplan)

differs

range or

coherence

20 24

(69)

Damon Runyan

214: "5:3"

indif. prof
265

motion/Rate

in.) required

2. a "prob."

82 65-68

(148) (83)

86

140-41

208

217

8P/P2

find

sources

2/multiple

my air/opnals

12-13-14

30

190 - B-H

(203 - wireframes)

71 - wireframes

108 183

72

50

Savage "trick"

38

108

242-43-44

248

260 - the fools brink!

is Good

Much

21

Risk,
and E

Brink

Daniel El

Garland P.
New York

KEY

108

145 141

38 142

108 97/165

72 197

181 "Amulet"

93 203

96 (258)

101 (P. H. H. H.)

173 264

188

types of
outgoing
sub, sequence
(+ in)

183

192

196 (4, 15)

231

spare, or
unintentional

Borel (1924)

111 (49)

Beyond Q+E

197 (intended)

Bayer-Herling)

"alluded to" - Q+E?

195

vs. "vagueness" as
equivalent to "indefiniteness"
of philosophy
probs.

(72) 14n

I say

Bayer-Herling, ignoring of outlying

is unreasonable (dangerous,
indefinite)

14; Keyes 13 Keyes 12

me 29

26

(262)

(217)

(206)

Bayer-Herling

is Good!

Meredith thing

21

R/T

reg. value

137, 157

200-209

217

249

257

PDA

prescription

decision

with

132

159

Risk, Ambiguity and Decision

Book

Daniel Ellsberg

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in objective

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Introduction

xvii

Ellsberg insisted that reasonable agents often refuse to be committed to numerically determinate probability distributions over states or even to complete orderings of states with respect to probabilities. As a consequence, they could not, under all circumstances, be expected utility maximizers.

Ellsberg's critique of Savage's approach (for which he registered profound admiration) has three prongs.

The first prong points to work by authors such as Koopman (1940) and Good (1952, 1962) who proposed formal theories of probability that allow for interval-valued probability judgment based on the testimony of deliberating agents concerning their comparative judgments of probability in addition to their testimony concerning their choice behavior in actual and hypothetical contexts of choice. Ellsberg is aware that Savage is a proponent of the first type. But Ellsberg insisted on the "meaningfulness" of judgments of comparative probability of the nonbehavioralist "verbalistic" variety. He did not claim that such testimony was sufficient to establish the coherence of judgments of probability that are not determinate. He insisted only that they not be discounted as meaningless.

The second prong of Ellsberg's argument appeals to Smith's account of upper and lower probabilities (Smith 1961), which purports to invoke only data of the "behavioralist" variety to elicit such probabilities. We are invited to entertain the suggestion that the interval-valued probabilities of Koopman and Good coincide with the interval values determined by Smith's upper and lower "pignic" probabilities. *or maybe not?*

If Savage's postulates are endorsed in their entirety as compelling conditions on rational agents, this "softening up" exercise will prove to be in vain. The third prong in Ellsberg's argument attempts to show that reasonable agents endorse choice behavior that violates the Savage postulates and that many such agents refuse to change their judgments when the fact of their violation is brought to their attention.

utility pairs. So it is misleading to claim that Savage's postulates determine a unique probability distribution over states, as Ellsberg suggests. What is true is that if one stipulates that the utility function must be state-independent (so that the utility of a given payoff is independent of the state as well as the act), there is exactly one probability that can serve in the representation. The postulation of state-independent utility cannot be formulated as a constraint on preferences over potential options of the sort that Savage calls "acts."

True?

* in '62 & added Y^0 and y^0 and p

and make X explicit.

from '61

THE SURE THING PRINCIPLE

Ellsberg undertook to undermine the persuasiveness of one of Savage's postulates: P2. Ellsberg called this the "Sure Thing Principle."⁷ I shall do the same. *Savage*

The content of P2 will be explained by considering the predicament of a decision-maker contemplating a (conceivably hypothetical) choice between options I and II in Figure 2, and also a choice between the options III and IV in Figure 3:

actually: a + b *Figure 2* *If they are "sure outcomes" (like c, d) this is just a decision*

are an "lotteries"

	H ₁	H ₂
Option I	a	c
Option II	b	c

to sure things

Figure 3

	H ₁	H ₂
Option III	a	d
Option IV	b	d

The sole difference between the decision problem depicted in Figure 2 and the decision problem depicted in Figure 3 concerns the payoffs for H₂. In the Figure 2 predicament, prize c is obtained regardless of how the decision-maker chooses. In Figure 3 predicament, the prize is d instead of c. Prizes a and b are the same in both cases.

Savage's P2 stipulates that Option I should be preferred to Option II if and only if Option III is preferred to Option IV. Changing the utility values for the payoff to be received in case H₂ is true should make no difference to the preference for the pair of options. As formulated here, P2 applies to problems where there are two states. The principle can be generalized to apply to any system of states determining finitely many distinct payoffs.

The Bernoulli proposition presupposes satisfaction of P2 along with several other postulates including in particular the assumption P1. P1 is an assumption that Savage's neo Bernoullian theory shares with the ordinalist theorists of consumer demand and with von Neumann and Morgenstern. It asserts that preferences over potential options yield a weak ordering.⁸

⁷According to Savage (1954, p. 24), P2 captures a presupposition of "the first part" of the Sure Thing Principle. The second part is captured by P3. It has, however, become widespread practice to consider P2 to be the Sure Thing Principle.

⁸The relation "x is weakly preferred to y" (or "x is preferred at least as

At the beginning of Chapter 8 (the last chapter of his dissertation), Ellsberg summarizes his case for the Restricted Bayes/Hurwicz Criterion as follows:

In the last two chapters the following testable propositions have been advanced: (1) certain information states can be meaningfully identified as ambiguous; (2) in these states, many otherwise reasonable people tend to violate the Savage axioms with respect to certain choices; (3) their behavior is deliberate and not readily reversed upon reflection; (4) certain patterns of "violating" behavior can be distinguished and described in terms of a specified decision rule; (5) this decision rule is one which has, in the past, been recommended or suggested as appropriate by reputable theorists; (6) many of the people concerned recognize the rule as reflecting the sort of considerations that influence their behavior and the rough character of actual mental processes by which they arrive at their decisions.

The discussion in chapters 6 and 7 that culminates in the proposal and elaboration of the Bayes/Hurwicz Criterion is a sustained effort to support these claims. By invoking a wide variety of problems and delivering verdicts that reasonable agents *might* deliver, Ellsberg argued for his Restricted Bayes/Hurwicz Criterion as the one that manages to accommodate the varieties of choice behavior that should be recognized as "reasonable" or "rational."¹⁴

As Ellsberg acknowledged, however, his critique of the personalist or "neo-Bernoullian" point of view, given exquisite expression in the axiomatic theory of Savage, might itself yield recommendations that "otherwise reasonable" people tend to violate. In Chapter 8, Ellsberg took up some challenges to the Bayes/Hurwicz Criterion and sought to respond to them. ?

INDETERMINACY IN PROBABILITY AND THE VON NEUMANN-MORGENSTERN AXIOMS.

A main theme of Ellsberg's dissertation is that failures of the Sure Thing Principle (or Savage's P2) arise in contexts where indeterminacy is present in subjective probability judgment. The famous three-color urn problem, often called "Ellsberg's paradox," is adduced in support of this idea. And Ellsberg's argument for the Bayes/Hurwicz Criterion is based on this problem and many other kinds of problems where indeterminacy or "ambiguity" arises in probability judgment.

¹⁴In his doctoral dissertation published one year after Ellsberg's, Nozick discussed Ellsberg's Restricted Bayes/minimax rule suggested in his 1961 paper. After registering some interesting doubts about this rule, Nozick suggested, in effect, that the Restricted Bayes/Hurwicz Criterion would be an improvement. Nozick remained skeptical at the end of this discussion as to the merits of this

i.e. in ~~same~~ year ^{after} as this:
1963: without having seen it!

of preference. He refused to countenance the possibility that P1 rather than P2 ought to be rejected. In this respect, his approach shares much in common with many of those who take indeterminacy in probability judgment seriously.

However, Ellsberg himself acknowledged that when a decision-maker lacks a determinate probability distribution over states but fails to rule out any distribution in a set Y^0 , to any available option "there will correspond, in general, a set of expected utility values, among which he cannot discriminate in terms of 'definite' probabilities" (p. 346). It is but one very short step from this observation to the conclusion that, when evaluating all of the available options with respect to expected utility, the decision-maker cannot weakly order them. If preference is to agree with expected utility, as a "neo-Bernoullian" would require, P1 will be violated. This must be true in Ellsberg's three-color urn problem. *But if often refer to violations of P1!*

If that is so, it is no longer clear that those who choose I over II and IV over III are violating P2, the Sure Thing Principle. That principle is a constraint on preference for potential options or acts. Such preference could be interpreted as preference with respect to expected utility.

When all expected utility functions that are "permissible" because they are derived using permissible probabilities in Y^0 agree in ranking option x over option y, then option x is strictly preferred to option y with respect to expected utility. If all permissible expected utility functions rate x as good as y, x is weakly preferred to y with respect to expected utility. If all permissible expected utility functions rank x and y as equal, x is equipreferred to y. When none of these conditions obtain, x and y are noncomparable with respect to expected utility.

Thus in the three-color urn problem, it is false that I is strictly preferred to II or IV to III with respect to expected utility. In point of fact I and II (III and IV) are noncomparable with respect to expected utility. So there cannot be a violation of P2 as long as preference is understood as preference with respect to expected utility. *and if P2 is violated!*

Preference, however, is often understood differently. Define binary revealed weak preference so that x weakly prefers (as revealed in binary choice) option 1 to option 2 if and only if in a choice between 1 and 2, x is prepared to choose 1. x strictly prefers 1 over 2 if and only if x weakly prefers 1 to 2 and does not weakly prefer 2 to 1. In that case, when confronted with a choice between 1 and 2, x is prepared to choose 1 but not 2. If weak preference holds in both directions, x equiprefers 1 and 2 (x is indifferent between 1 and 2).

According to this criterion, weak preference is complete by definition as long as we suppose that x is always "prepared" to choose at least one of the options in binary choice. It is logically possible that a rational decision-maker who finds options I and II to be noncomparable with respect to expected utility could find I strictly preferred to II in the sense of binary revealed preference. Similarly III and IV could be noncomparable with

*No - for Savage, proves + expected utility
exist $\Rightarrow$ binary choice only P-2 (not others);
As P2 is alleged in "rational pref." then...*

not) without such reinforcing.

For all its introductory tone, it is really to the currently convinced—of opposing view—that this study is most directly addressed. The readers whose anticipated responses have most influenced the inclusion of otherwise redundant material are those I have characterized in the text as the “neo-Bernoullians.” Very simply: I wish to change their minds. I hope to convince them of the importance of certain questions and problems, of the need and possibility for including in their formal theories certain considerations now explicitly ignored; and to persuade them there are more ways of being reasonable under uncertainty than they currently imagine.

^{be > 30 years}
 & failed—except to reveal
 to the that forms our “rational,”
 in more ways than they feel addressed.
 (and even they were tempted).

Then resolution of cognitive
 dissonance:

4/they are, generally, rational,
 and want to be, all the time. But in
 these cases they do rules that, to not
 and want, define rationality; and don't
 want to change when this is pointed out.
 (Even & are tempted...)

gain!

Thus, Keynes, in *A Treatise on Probability* (1921), introduced formally the notion of non-comparability of beliefs.

Is our expectation of rain, when we start out for a walk, always *more* likely than not, or *less* likely than not, or *as* likely as not? I am prepared to argue that on some occasions *none* of these alternatives hold . . . If the barometer is high but the clouds are black, it is not always rational that one should prevail over the other in our minds, or even that we should balance them.¹

However, as Keynes later points out, while it may be impossible for a given person on a given occasion to compare definitely the probability of "rain" to that of "not-rain," it may be possible to compare each of these to the probabilities of certain *other* propositions, to which it may even be possible to assign numerical values (by processes we have not yet considered); it may then be possible to make meaningful statements of the form, "the probability of 'rain' for me is greater than that of proposition X, which has probability = .3, and less than of proposition Y, which has probability = .7." Unless the resulting inequalities confine the probability of 'rain' "within narrow numerical limits," Keynes tends to say that a probability "does not exist,"² rather than that it is "unknown," but I. J. Good objects to both phrases, preferring the point of view that:

The initial probabilities may always be assumed to exist within the abstract theory, but in some cases you may be able to judge only that they lie in rather wide intervals. This does not prevent the application of Bayes' theorem: it merely makes it less effective than if the intervals are narrow.

It is hardly satisfactory to say that the probabilities do not exist when the intervals are wide, while admitting that they do exist when the intervals are narrow.³

To suppose that initial probabilities, in problems of inference, can always be judged precisely is an example of the precision fallacy, Good asserts. He emphasizes that most judgments of relative probability are in the form of inequalities, A "more probable than" B, rather than precise equalities; a body of judgments of this form can be described numerically, but the description is generally in terms of intervals of probability numbers, circumscribed by inequalities (the process by which Koopman and Good

¹John Maynard Keynes, *A Treatise on Probability* (London, 1952), p. 30.

²Keynes, *op. cit.*, p. 31.

³Good, *Probability and the Weighing of Evidence*, p. 40.

* behaviorally-inferable ("revealed")

"probabilities" do not exist.

or "intervals" are known/exist only with intervals.

JK doesn't state "intervals"!

YES

*

Hypo: worst decisions. IJG doesn't allow Keynes' crit.

does IJG apply them to decisions?

only to inferences?

*I offer several reasonable
rules: (not just rational
minimization: as in QJE)*

13

amount of confidence in that opinion as it does upon the favorableness of the opinion itself.¹

But *how* does it depend? *How* may the web of action systematically reflect the varying degrees of "vagueness" of "ambiguity/weight," of "confidence" in our judgment? On that question, which is of central interest in this study, Knight is virtually silent, as are Savage and Keynes (Good makes several conjectures, identical to some examined in Chapters Six and Seven). It is a puzzle, they agree.² Knight offers the comment: "The ultimate logic, or psychology, of these deliberations is obscure, a part of the scientifically unfathomable mystery of life and mind."³ But where the recent "neo-Bernoullians" tend to shrug their shoulders at the difficulties of expressing these factors and measuring their influence, effectively ignoring them in their formal theorizing, Keynes, like Knight, emphasizes that these matters do seem relevant to decision-making, though admitting frankly his own vagueness and lack of confidence on this particular question:

In deciding on a course of action, it seems plausible to suppose that we ought to take account of the weight as well as the probability of different expectations. But it is difficult to think of any clear example of this, and I do not feel sure that the theory of 'evidential weight' has much practical significance.⁴

If two probabilities are equal in degree, ought we, in choosing our course of action, to prefer that one which is based on a greater body of knowledge?

The question appears to me to be highly perplexing, and it is difficult to say much that is useful about it. But the degree of completeness of the information upon which a probability is based does seem to be relevant, as well as the actual magnitude of the probability, in making practical decisions . . . If, for one alternative, the available information is necessarily small,

¹Knight, *op. cit.*, pp. 226-227.

²"So we ask, but we seldom receive an answer: the [probability theorists] have turned away, and like antique heroes we only know that we have been talking with the immortals from the fact that they are no longer there." (H. R. Trevor-Roper, "The Mind of Adolf Hitler," introduction to *Hitler's Secret Conversations*, New York, 1953, p. vii) [paraphrased].

³*Ibid.*, p. 227.

⁴*Op. cit.*, p. 76. Keynes is equally apologetic in introducing his chapter on "The Weight of Arguments": "The question to be raised in this chapter is somewhat novel; after much consideration I remain uncertain as to how much importance to attach to it." (*Ibid.*, p. 71.)

into the utility, reasonable": individual each reactions or accept-

und and position" ions, for in some ind, bet-ries that ement of defined) mpass a nportant decision- nstances nsistent vill often can find les.¹ price of tical cal-ieve that below-gambles

an order- of utility stern; yet ther con- to be pos-ies," and s XII and ork, 1954, re-Thing

ent on the inciple

in terms of precise probabilities and utilities, "while, in practice, people with common sense" take reasonable account, in their behavior, of their degree of ignorance of both

It is true that the vulgar evaluations I seek to "rationalize" encounter closely-reasoned and persuasive arguments that they violate "self-evident" principles of reasonable decision-making. Yet I am encouraged to challenge what is becoming a new orthodoxy of opinion by the thought that the theory of expected value defeated by Cramer and Bernoulli—so easily, it now seems to us, with their introspective exercises—appeared to generations of subtle minds as just as unshakable, as intuitively and logically compelling, as uniquely "right," as does the "Sure-Thing Principle" today to the truest believer.¹

To act on random things: all (or most!) cases is understandable for many / most subjects (and) violates are not only "not-rules / irrational" but more reasonable (non-violates may be / look more) "a single source" (for journalists: esp. a single non-official source (esp. without a doc); supposedly less reliable, unworthily (esp. when contradicting an official source) than a single official source. Actually, one wanted source can be worth 10 officials (all dependent on a boss, or Paris (e.g. "war is justified; for following reasons / benefits" ")).

Can't just count the number of sources (e.g. Iraqi defectors; Eke officials... kept back info; *

was at first so categorically accepted that it seemed paradoxical to mathematicians of the early eighteenth century that presumably prudent individuals reject the principle in certain real and hypothetical situations." (The Foundations of Statistics, p. 92.)

¹ I am indebted to L. J. Savage for suggesting this analogy, in conversation (which is not to say that his assessment of its import is identical with mine).

* Tom Wells' sources!